CALCULUS.

NOTE ON PROBLEM 84, BY DR. E. WOELFFING, STUTTGART, GERMANY.

The solution of question 84 can be found in some theorems proved by W. Merkelbach (*Ueber Rollkurven welche von einer Graden eingehüllt werden*, Diss. Marburg, 1881). The first of them is the following:

If a curve, C, rolls upon another curve, C', and a point, P, in the plane of C describes a straight line L, and we make afterwards the curve C' roll upon the curve C, then a straight line, L, in the plane of C' will always pass through P (page 18 of the paper quoted).

Now (and this is the second theorem of Merkelbach) if a sinusoid rolls upon an ellipse, a straight line in the plane of the former passes through a focus of the latter (page 24); therefore, if the ellipse rolls upon the sinusoid, any one of the foci of the former will describe a straight line.

Stuttgart, Germany, July 19, 1899.

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91. Proposed by GUY B. COLLIER, Schenectady, N. Y.

Find the area of a loop of the curve $r^2\cos\theta=a^2\sin3\theta$. [From Hall's *Differential and Integral Calculus*].

I. Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; M. C. STEVENS, A. M., Professor of Mathematics, Purdue University, Lafayette, Ind.; WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; ELMER SCHUYLER, Reading, Penna.; and J. SCHEFFER, A. M., Hagerstown, Md.

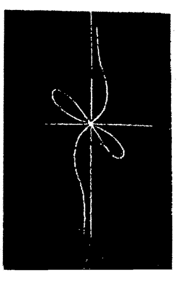
The curve has two equal loops, one in the first and the other in the third quadrant.

The limits of θ are 0 and $\frac{1}{3}\pi$.

$$\therefore\ A=\tfrac{1}{2}\int_0^{\frac{1}{3}\pi} r^2\,d\theta=\tfrac{1}{2}a^2\int_0^{\frac{1}{3}\pi}\frac{\sin3\theta\,d\theta}{\cos\theta}$$

$$=\tfrac{1}{2}a^2\int_0^{\frac{1}{3}\pi}(4\sin\theta\cos\theta-\tan\theta)\,d\theta=\tfrac{1}{4}a^2(3-2\log2).$$

II. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; and GEORGE LILLEY, Ph. D., Professor of Mathematics, University of Oregon, Eugene, Ore.



The shape of the curve is seen in the diagram.

The limits are evidently from 0° to 30° for a loop.

$$\text{Then } A=\tfrac{1}{2}\int r^2\,d\theta=\tfrac{1}{2}\int_0^{\frac{1}{6}\pi}\frac{a^2\sin3\theta}{\cos\theta}d\theta$$

$$=\frac{a^2}{2}\int_0^{\frac{1}{6}\pi}\frac{3\sin\theta-4\sin^3\theta}{\cos\theta}d\theta=\frac{a^2}{2}\int_0^{\frac{1}{6}\pi}3\tan\theta\,d\theta$$

$$-2a^2\int_0^{\frac{1}{6}\pi}(\tan\theta-\sin\theta\cos\theta)\,d\theta$$